List of publications included in the Thesis and the main contributions:

- Appendix A Value Iteration Solver Networks, International Conference on Intelligent Autonomous Systems, 2020, IEEE, Evalds Urtans, Valters Vecins. Introduced a novel model UNet-RNN-Skip for improving the performance of the Value Iteration Algorithm and a novel synthetic dataset generator OccupancyMapGenerator for evaluation of path planning models.
- Appendix B Software-Assisted Method Development in High Performance Liquid Chromatography; ISBN: 978-1-78634-545-5, Sep. 2018, Sergey V. Galushko, Irina Shishkina, Evalds Urtans, Oksana Rotkaja. Introduced a novel Deep Reinforcement Learning based method for sequentially developing solvent gradients in HPLC.
- Appendix C Survey of Deep Q-Network variants in PyGame Learning Environment, International Conference on Deep Learning technologies, 2018, ACM, Evalds Urtans, Agris Nikitenko Introduced a novel Deep Reinforcement Learning based method and a novel MDQN loss function.
- Appendix D Exponential Triplet Loss, International Conference on Compute and Data Analysis, 2020, IEEE/ACM, Evalds Urtans, Valters Vecins, Agris Nikitenko Introduced a novel Deep Metric Learning based loss function Exponential Triplet Loss.
- Appendix E asya: Mindful verbal communication using deep learning, Cornell University, Computing Research Repository, 2020, Evalds Urtans, Austris Tabaks Introduced a novel system based on Exponential Triplet Loss for voice reidentification.

## 2 LITERATURE REVIEW

This section aims to provide a background of existing research in the loss functions of the Deep Metric Learning. It explains the importance of the research and activity in the development of novel loss functions. Similar survey of methods and loss functions has been done also for Deep Reinforcement Learning [109]. Results of the findings for Deep Reinforcement Learning have been included in Appendix C.

## 2.1 Methodolgy of Literature Review

The methodology of SLR (Systematic Literature Review) presented in this document is based on a systematic mapping study [80] [43]. The results of SLR contain the map of clusters based on the origins of loss functions and methods, as well as a qualitative review based on research questions. The results also include a list of limitations identified for loss functions and methods used in the reviewed papers.

The method for selecting and evaluating papers contains the steps listed in Fig. 3. Initially the most well-known publications [7], [23] in the field of deep metric learning (DML) have been selected. Additionally, the following keywords were used for the initial search of papers: triplet loss, contrastive loss, ranking loss, deep metric learning, representation learning, oneshot learning, zero-shot learning, product re-identification task, signature re-identification, face re-identification task. Then the publications have been thoroughly analyzed and documented to check if publications match the field of DML loss function research. Then matching to Quality Assessment criteria has been evaluated. If at least single assessment criterion has been met, a publication was added to the main list. In addition, if answers to research questions have been found in selected publications, then those were documented. The references and citations of this publication have been found. For each of the relevant publications, their citation count has been found and divided by years passed since publishing. Those with the highest value of influence were analyzed first.

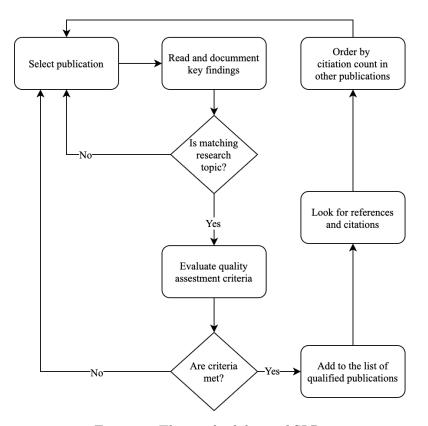


Figure 3: The methodology of SLR.

To find a valid direction of further research, few research questions (RQ) were selected. The research questions addressed by this study are:

- RQ1: What kinds of functions have been studied similar to Triplet Loss functions?
- RQ2: Do the novel loss functions achieve significantly better results than previous functions?
- RQ3: Do the novel loss functions have theoretical grounding, or are they purely empirical?
- RQ4: What are the limitations of novel loss functions?

## 2.2 Results of the Literature Review on Deep Metric Learning

The results of SLR regarding DML are mapped in multiple tables depending on the relevant properties extracted from papers. Information about authors, affiliation, country of origin, and conferences regarding DML have been listed in Table 1.

Publications have been ordered by the year of publishing, and the numbering of publications has been maintained also in the following tables.

 $\label{eq:Table 1:} \mbox{ Table 1:}$  Authors and conferences on studies regarding DML.

| No | Title  | Authors   | Affiliation  | Country | Year | Conference / Journal       |
|----|--|---|--|---------|------|----------------------------|
| 1  | Signature Verification Us-<br>ing A "Siamese" Time De-<br>lay Neural Network [7]           | J. Bromley, J. Bentz, L. Bottou, I. Guyon, Y. LeCun, C. Moore, E. Sckinger, R. Shah | AT&T Bell laboratories   | USA     | 1993 | INT J<br>PATTERN<br>RECOGN |
| 2  | Neighbourhood Components Analysis [27]   | J. Goldberger,<br>S. Roweis,<br>G. Hinton,<br>R. Salakhutdinov                      | AT&T Bell laboratories   | Canada  | 2004 | NIPS                       |
| 3  | Learning a Similarity Metric Discriminatively, with Application to Face Verification [14]  | S. Chopra,<br>R. Hadsell,<br>Y. LeCun   | NYU  | USA     | 2005 | CVPR                       |
| 4  | Distance metric learning<br>for large margin near-<br>est neighbor classification<br>[127] | K. Q. Weinberger,<br>L. Saul  | Yahoo!,<br>University of Cal-<br>ifornia                                       | USA     | 2005 | NIPS                       |
| 5  | Large scale metric learning from equivalence constraints [46]                              | M. KŽstinger, M. Hirzer, P. Wohlhart, P. Roth, H. Bischof                           | Graz University<br>of Technology   | Austria | 2012 | CVPR                       |
| 6  | Quadruplet-Wise Image<br>Similarity Learning [52]  | M. Law,<br>N. Thome,<br>M. Cord   | Sorbonne University  | France  | 2013 | ICCV                       |
| 7  | Reidentification by Relative Distance Comparison [135]                                     | W. Zheng,<br>S. Gong,<br>T. Xiang   | College of Electronic and Information,<br>South China University of Technology | China   | 2013 | TPAMI                      |
| 8  | Deep Metric Learning<br>for Practical Person<br>Re-Identification [132]                    | D. Yi,<br>Z. Lei,<br>S. Li  | IEEE   | China   | 2014 | ArXiv                      |
| 9  | FaceNet: A unified<br>embedding for face<br>recognition and<br>clustering [23]             | F. Schroff, D. Kalenichenko, J. Philbin   | Google   | USA     | 2015 | CVPR                       |
| 10 | Improved Deep Metric<br>Learning with Multi-class<br>N-pair Loss Objective [98]            | K. Sohn   | NEC  | USA     | 2016 | NIPS                       |
| 11 | A Discriminative Feature<br>Learning Approach for<br>Deep Face Recognition<br>[128]        | Y. Wen,<br>K. Zhang,<br>Z. Li,<br>Y. Qiao   | SIAT   | China   | 2016 | ECCV                       |

| 12  | Deep Metric Learning via                         | H. O. Song,            | Stanford Univer-           | USA      | 2016 | CVPR   |
|-----|--|------------------------|----------------------------|----------|------|--------|
| 12  | Lifted Structured Feature                        | Y. Xiang,              | sity,                      | USA      | 2010 | CVFR   |
|     | Embedding [100]                                  | S. Jegelka,            | MIT                        |          |      |        |
|     | Embedding [100]                                  | S. Savarese            | NII I                      |          |      |        |
| 13  | Deep   | J. Hershey,            | Mitsubishi,                | USA      | 2016 | ICASSP |
| -   | clustering: Discriminative                       | Z. Chen,               | Columbia Univer-           |          |      |        |
|     | embeddings for segmenta-                         | J. Le Roux,            | sity                       |          |      |        |
|     | tion and   | S. Watanabe            |                            |          |      |        |
|     | separation [34]                                  |                        |                            |          |      |        |
| 14  | Learning Deep Embed-                             | E. Ustinova,           | Skoltech                   | Russia   | 2016 | NIPS   |
|     | dings with Histogram                             | V. Lempitsky           |                            |          |      |        |
|     | Loss [113]                                       |                        |                            |          |      |        |
| 15  | Local Similarity-Aware                           | C. Huang,              | The Chinese Uni-           | China    | 2016 | NIPS   |
|     | Deep Feature Embedding                           | C. C. Loy,             | versity of Hong            |          |      |        |
|     | [37]   | X. Tang                | Kong,                      |          |      |        |
|     |  |                        | SenseTime Group<br>Limited |          |      |        |
| 16  | Metric Learning with                             | O. Rippel,             | Facebook                   | USA      | 2016 | ICLR   |
| 10  | Adaptive Density Dis-                            | M. Paluri,             | racebook                   | USA      | 2016 | ICLK   |
|     | crimination [86]                                 | P. DollĞr,             |                            |          |      |        |
|     | crimination [60]                                 | L. D. Bourdev          |                            |          |      |        |
| 17  | L2-constrained Softmax                           | R. Ranjan,             | UMIACS                     | USA      | 2017 | ArXiv  |
| l   | Loss for Discriminative                          | C. D. Castillo,        |                            |          | 1    |        |
|     | Face Verification [84]                           | R. Chellappa           |                            |          |      |        |
| 18  | In Defense of the Triplet                        | A. Hermans,            | RWTH                       | Germany  | 2017 | ArXiv  |
|     | Loss for Person Re-                              | L. Beyer,              |                            |          |      |        |
| 1   | Identification [33]                              | B. Leibe               |                            |          |      |        |
| 19  | Deep Metric Learning                             | J. Wang,               | Baidu                      | China    | 2017 | ICCV   |
|     | with Angular Loss [117]                          | F. Zhou,               |                            |          |      |        |
|     |  | S. Wen,                |                            |          |      |        |
|     |  | X. Liu,                |                            |          |      |        |
|     |  | Y. Lin                 |                            |          |      |        |
| 20  | No Fuss Distance Met-                            | Y. Movshovitz-         | Google                     | USA      | 2017 | ICCV   |
|     | ric Learning Using Proxies                       | Attias,                |                            |          |      |        |
|     | [70]   | A. Toshev,             |                            |          |      |        |
|     |  | T. Leung,<br>S. Ioffe, |                            |          |      |        |
|     |  | S. Singh               |                            |          |      |        |
| 21  | Sampling Matters in Deep                         | R. Manmatha,           | UT Austin.                 | USA      | 2017 | ICCV   |
| 21  | Embedding Learning [64]                          | C. Y. Wu,              | Amazon                     | UDA      | 2017 | 100 V  |
|     | Embedding Ecarning [04]                          | A. Smola,              | Timazon                    |          |      |        |
|     |  | P. KrŁhenb§hl          |                            |          |      |        |
| 22  | Deep Metric Learning via                         | H. O. Song,            | Google                     | USA      | 2017 | CVPR   |
|     | Facility Location [99]                           | S. Jegelka,            |                            |          |      |        |
|     |  | V. Rathod,             |                            |          |      |        |
|     |  | K. Murphy              |                            |          |      |        |
| 23  | Deep spectral clustering                         | M. Law,                | University of              | Canada   | 2017 | ICML   |
|     | learning [53]                                    | R. Urtasun,            | Toronto                    |          |      |        |
|     |  | R. Zemel               |                            |          |      |        |
| 24  | Hard-Aware Deeply Cas-                           | Y. Yuan,               | MOE,                       | China    | 2017 | ICCV   |
| 1   | caded Embedding [133]                            | K. Yang,               | Peking Univer-             |          |      |        |
|     |  | C. Zhang               | sity,                      |          |      |        |
|     |  |                        | DeepMotion,                |          |      |        |
|     |  |                        | Microsoft Re-<br>search    |          |      |        |
| 25  | PPFNet: Global Context                           | H. Deng,               | TMU,                       | Germany, | 2018 | CVPR   |
| ∠3  | Aware Local Features                             | H. Deng,<br>T. Birdal, | NUDT                       | China    | 2018 | CVFR   |
|     | for Robust 3D Point                              | S. Ilic                |                            |          |      |        |
|     | Matching [17]                                    |                        |                            |          |      |        |
| 26  | Ranked List Loss for Deep                        | X. Wang,               | Anyvision,                 | UK       | 2019 | CVPR   |
|     | Metric Learning [121]                            | Y. Hua,                | QueenÕs Univer-            |          |      |        |
| 1   |  | E. Kodirov,            | sity Belfast               |          |      |        |
|     |  | G. Hu,                 | _                          |          |      |        |
|     |  | R. Garnier,            |                            |          |      |        |
|     |  | N. Robertson           |                            |          |      |        |
| 27  | Multi-Similarity Loss with                       | X. Wang,               | Malong Technolo-           | China    | 2019 | CVPR   |
| 1   | General Pair Weighting                           | Xintong Han,           | gies                       |          |      |        |
|     | for Deep Metric Learning                         | W. Huang,              |                            |          |      |        |
|     | [119]  | D. Dong,               |                            |          |      |        |
| 0.0 | A C: 1 727                                       | M. Scott               | mi cii: **:                | CI :     | 0010 | P.C.C. |
| 28  | A Simple and Effective<br>Framework for Pairwise | Q. Qi,<br>Y. Yan,      | The Chinese Uni-           | China    | 2019 | ECCV   |
| 1   | Deep Metric Learning [82]                        |                        | versity of Hong<br>Kong    |          |      |        |
| 1   | Deep Metric Learning [82]                        | Z. Wu,<br>X. Wang,     | Trong                      |          |      |        |
|     | 1  | A. wang,               | I                          | I        | 1    | 1      |
|     |  | T. Yang                |                            |          |      |        |

| 29 | Deep Metric Learning     | B. X. Nguyen, | AIOZ | Singapore | 2020 | ArXiv |
|----|--------------------------|---------------|------|-----------|------|-------|
|    | Meets Deep Clustering:   | B. D. Nguyen, |      |           |      |       |
|    | An Novel Unsupervised    | G. Carneiro,  |      |           |      |       |
|    | Approach for Feature     | E. Tjiputra,  |      |           |      |       |
|    | Embedding [73]           | Q. D. Tran,   |      |           |      |       |
|    |                          | T. T. Do      |      |           |      |       |
| 30 | Exponential triplet loss | E. Urtans,    | RTU  | Latvia    | 2020 | ICCDA |
|    | [110]                    | A. Nikitenko, |      |           |      |       |
|    |                          | V. Vecins     |      |           |      |       |

In, Table 3 information about novel loss functions and their properties regarding DML have been listed. Embedding space refers to normalization or measurement methods between two or more vectors in a latent space. Each of the embedding vectors has been produced by a deep learning based model for the data point. Then two or more embedding vectors have been processed using the loss function and a deep learning based model weights are calculated using the back-propagation algorithm. In addition, for many of these papers sample mining methods are used to select the best training samples to improve the results and speed of the training.

 $\label{eq:Table 3:} Table \ 3:$  Novel loss functions of studies regarding DML.

| No | Title  | Year | Embedding space             | Sample Mining   | Loss function                             |  |
|----|--|------|-----------------------------|-----------------|---|--|
| 1  | Signature Verification Us-<br>ing A "Siamese" Time De-<br>lay Neural Network [7]           | 1993 | Euclidean                   | None            | Contrastive loss                          |  |
| 2  | Neighbourhood Compo-<br>nents Analysis [27]  | 2004 | Euclidean, Maha-<br>lanobis | None            | NCA Loss                                  |  |
| 3  | Learning a Similarity Metric Discriminatively, with Application to Face Verification [14]  | 2005 | L1, Euclidean               | None            | Contrastive Loss                          |  |
| 4  | Distance metric learning<br>for large margin near-<br>est neighbor classification<br>[127] | 2005 | Euclidean, Maha-<br>lanobis | None            | Triplet Hinge<br>Loss                     |  |
| 5  | Large scale metric learn-<br>ing from equivalence con-<br>straints [46]                    | 2012 | Mahalanobis                 | None            | KISS-BCE Loss                             |  |
| 6  | Quadruplet-Wise Image<br>Similarity Learning [52]  | 2013 | Qwise                       | None            | Quadruplet<br>Hinge Loss                  |  |
| 7  | Reidentification by Relative Distance Comparison [135]                                     | 2013 | RDC                         | None            | RDC Loss                                  |  |
| 8  | Deep Metric Learning<br>for Practical Person<br>Re-Identification [132]                    | 2014 | Cosine distance             | Hard            | Binomial De-<br>viance Loss               |  |
| 9  | FaceNet: A unified<br>embedding for face<br>recognition and<br>clustering [23]             | 2015 | L2, Euclidean               | Hard, Semi-Hard | Triplet Loss,<br>Harmonic Triplet<br>Loss |  |
| 10 | Improved Deep Metric<br>Learning with Multi-class<br>N-pair Loss Objective [98]            | 2016 | L2, Cosine distance         | N Hard Mining   | multi-class N-pair<br>loss                |  |
| 11 | A Discriminative Feature<br>Learning Approach for<br>Deep Face Recognition<br>[128]        | 2016 | Cosine distance             | None            | Center loss                               |  |

| 12  | Deep Metric Learning via                           | 2016 | L2, Euclidean   | Mining positives            | Lifted Structured                    |
|-----|--|------|-----------------|-----------------------------|--------------------------------------|
|     | Lifted Structured Feature                          |      |                 |                             | Loss,                                |
| 13  | Embedding [100] Deep                               | 2016 | L2. Euclidean   | None                        | Lifted Struct Pairwise metric        |
| 13  | clustering: Discriminative                         | 2016 | L2, Euchdean    | None                        | Loss metric                          |
|     | embeddings for segmenta-                           |      |                 |                             | 1033                                 |
|     | tion and   |      |                 |                             |                                      |
|     | separation [34]                                    |      |                 |                             |                                      |
| 14  | Learning Deep Embed-<br>dings with Histogram       | 2016 | Cosine distance | None                        | Histogram Loss                       |
|     | Loss [113]   |      |                 |                             |                                      |
| 15  | Local Similarity-Aware                             | 2016 | PDDM            | Hard mining                 | PDDM - Double                        |
|     | Deep Feature Embedding                             |      |                 |                             | Header Hinge                         |
| 4.0 | [37]   | 2012 |                 | 1                           | Loss                                 |
| 16  | Metric Learning with<br>Adaptive Density Dis-      | 2016 | Euclidean       | Neighbourhood Sam-<br>pling | Magnet Loss                          |
|     | crimination [86]                                   |      |                 | pinig                       |                                      |
| 17  | L2-constrained Softmax                             | 2017 | Cosine distance | None                        | L2 constrained                       |
|     | Loss for Discriminative                            |      |                 |                             | Softmax Loss                         |
| 1.0 | Face Verification [84]                             | 0017 | 10 5 1:1        | N                           | D 4 1 All (D : 1 4                   |
| 18  | In Defense of the Triplet<br>Loss for Person Re-   | 2017 | L2, Euclidean   | None                        | Batch All Triplet<br>Loss            |
|     | Identification [33]                                |      |                 |                             | LOSS                                 |
| 19  | Deep Metric Learning                               | 2017 | Angle           | None                        | Angular loss                         |
|     | with Angular Loss [117]                            |      |                 |                             |                                      |
| 20  | No Fuss Distance Met-                              | 2017 | L2, Euclidean   | None                        | Proxy Ranking                        |
|     | ric Learning Using Proxies [70]                    |      |                 |                             | Loss,<br>Proxy NCA Loss              |
| 21  | Sampling Matters in Deep                           | 2017 | L2, Euclidean   | Distance weighted           | Triplet Loss,                        |
|     | Embedding Learning [64]                            |      | <u> </u>        | sampling                    | Contrasitve Loss                     |
| 22  | Deep Metric Learning via                           | 2017 | L2, Euclidean   | None                        | Struct Clust,                        |
| 23  | Facility Location [99]  Deep spectral clustering   | 2017 | L2, Euclidean   | None                        | Clustering Loss<br>Spectral Cluster- |
| 23  | learning [53]                                      | 2017 | L2, Euchdean    | None                        | ing Loss                             |
| 24  | Hard-Aware Deeply Cas-                             | 2017 | Euclidean       | Model-based                 | Any / Con-                           |
|     | caded Embedding [133]                              |      |                 |                             | trastive loss                        |
| 25  | PPFNet: Global Context                             | 2018 | L2, Euclidean   | None                        | N-Tuple loss                         |
|     | Aware Local Features<br>for Robust 3D Point        |      |                 |                             |                                      |
|     | Matching [17]                                      |      |                 |                             |                                      |
| 26  | Ranked List Loss for Deep                          | 2019 | Euclidean       | Hard                        | Ranked List Loss                     |
|     | Metric Learning [121]                              |      |                 |                             |                                      |
| 27  | Multi-Similarity Loss with                         | 2019 | Cosine distance | Hard                        | Multi-Similarity                     |
|     | General Pair Weighting<br>for Deep Metric Learning |      |                 |                             | Loss                                 |
|     | [119] for Deep Metric Learning                     |      |                 |                             |                                      |
| 28  | A Simple and Effective                             | 2019 | Euclidean       | TopK Loss mining            | DRO-TopK Loss                        |
|     | Framework for Pairwise                             |      |                 |                             | -                                    |
| 20  | Deep Metric Learning [82]                          | 2020 | 10 E 1:1        | N                           | 77 . 1                               |
| 29  | Deep Metric Learning<br>Meets Deep Clustering:     | 2020 | L2, Euclidean   | None                        | Unsupervised<br>UDML Loss            |
|     | An Novel Unsupervised                              |      |                 |                             | ODMII 1088                           |
|     | Approach for Feature                               |      |                 |                             |                                      |
|     | Embedding [73]                                     |      |                 |                             |                                      |
| 30  | Exponential triplet loss                           | 2020 | Unit-Range      | Hard                        | Exponential                          |
|     | [110]  |      |                 |                             | Triplet Loss                         |

In Fig. 4 relationship of DML Loss functions has been summarized. Colours denote similar groups of loss functions by their origin and methodology. It is possible to observe that most of the loss functions come from the seminal works of Contrastive Loss [7], NCA Loss [27], and Triplet Loss [127]. Most of the functions are extensions of simple Hinge Loss [127]. As seen in Table 3, most of the loss functions use sample mining methods, because they are trained only using a few data samples per training iteration. Some methods like Histogram Loss [113] or Quadruplet Hinge Loss [52] use more samples per training iteration, but their results on benchmark datasets are not significantly better than other methods as seen in Table 5.

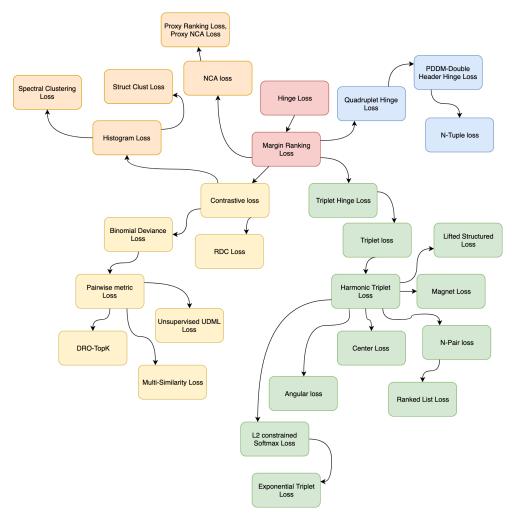


Figure 4: Relationship of DML Loss functions. Colours denote similar groups of loss functions by their origin and methodology.

Table 5 lists practical applications for each of DML loss functions that have been studied, as well as their benchmark datasets and the best results on those datasets. Where applicable, Top-1 accuracy has been selected for the best results on each of the datasets. As seen in the listings, most of the practical applications and datasets have been used for face and product re-identification.

 $\label{eq:Table 5: Table 5: Practical applications and best results for every dataset.}$ 

| No | Title   | Year | Practical<br>application                                   | Dataset / Top-1<br>Acc.  |
|----|---|------|--|--|
| 1  | Signature Verification Us-<br>ing A "Siamese" Time De-<br>lay Neural Network [7]              | 1993 | Signature re-<br>identification                            | Signatures: 97%  |
| 2  | Neighbourhood Components Analysis [27]  | 2004 | Handwriting iden-<br>tification, Face<br>re-identification | USPS: 85%<br>FERET-B   |
| 3  | Learning a Similarity Metric Discriminatively, with Application to Face Verification [14]     | 2005 | Face Re-identification                                     | AT&T: 92.5%  |
| 4  | Distance metric learning<br>for large margin near-<br>est neighbor classification<br>[127]    | 2005 | Handwriting identifi-<br>cation, text classifica-<br>tion  | MNIST: 98.8%<br>Letters: 96.3%<br>20news: 92%<br>Isolet: 96.6%<br>YaleFaces: 93.9%     |
| 5  | Large scale metric learning from equivalence constraints [46]                                 | 2012 | Face Reidentification, Image Reidenification               | LFW: 80.5%<br>VIPeR: 22%   |
| 6  | Quadruplet-Wise Image<br>Similarity Learning [52]   | 2013 | Product or image re-<br>trieval                            | OSR: 74.6%<br>Pubfig: 77.6%  |
| 7  | Reidentification by Relative Distance Comparison [135]  | 2013 | Face Re-identification                                     | ETHZ: 61.58%<br>i-LIDS: 32.60%<br>VIPeR: 9.12%   |
| 8  | Deep Metric Learning<br>for Practical Person<br>Re-Identification [132]                       | 2014 | Face Re-identification                                     | VIPER: 34.49%  |
| 9  | FaceNet: A unified<br>embedding for face<br>recognition and<br>clustering [23]                | 2015 | Face Re-identification                                     | LFW: 99.63%<br>YTF: 95.12%   |
| 10 | Improved Deep Metric<br>Learning with Multi-class<br>N-pair Loss Objective [98]               | 2016 | Product image retrieval, Face Re-identification            | LFW: 98.33%<br>SOP: 28.19%<br>CAR-196: 33.5%<br>CUB-200: 27.24%                        |
| 11 | A Discriminative Feature<br>Learning Approach for<br>Deep Face Recognition<br>[128]           | 2016 | Face Re-identification                                     | LFW: 99.28%<br>YTF: 94.9%<br>MegaFace: 76.5%   |
| 12 | Deep Metric Learning via<br>Lifted Structured Feature<br>Embedding [100]                      | 2016 | Product or image retrieval                                 | CUB200: 55%,<br>CARS196: 48%,<br>SOP: 62%  |
| 13 | Deep<br>clustering: Discriminative<br>embeddings for segmenta-<br>tion and<br>separation [34] | 2016 | Speaker diarization,<br>seperation                         | WSJ0: 2.74 dB (SDR)  |
| 14 | Learning Deep Embeddings with Histogram Loss [113]  | 2016 | Product or image re-<br>trieval                            | CUHK03: 65.7%<br>CUB-200: 51%<br>Market-1501:<br>59.47%<br>SOP: 65%                    |
| 15 | Local Similarity-Aware Deep Feature Embedding [37]  | 2016 | Product or image re-<br>trieval                            | CARS196: 57.4%<br>CUB-200: 58.3%<br>ImageNet: 48.2%                                    |
| 16 | Metric Learning with<br>Adaptive Density Dis-<br>crimination [86]                             | 2016 | Image classification, Face Re-identification               | Stanford Dogs: 75.1%<br>Flowers-102: 91.4%<br>Oxford-HIT Pet: 89.4%<br>ImageNet: 84.1% |
| 17 | L2-constrained Softmax<br>Loss for Discriminative<br>Face Verification [84]                   | 2017 | Image classification,<br>Face Re-identification            | LFW: 99.33%<br>YTF: 99.78%<br>MNIST: 99.05%<br>IJB-A: 97.5%                            |
| 18 | In Defense of the Triplet<br>Loss for Person Re-<br>Identification [33]                       | 2017 | Product image retrieval, Face Re-identification            | MARS: 90.53%,<br>Market-1501: 79.8%,<br>CUHK03: 87.58%                                 |

| 19 | Deep Metric Learning<br>with Angular Loss [117]   | 2017 | Product or image re-<br>trieval                            | CAR-196: 71.4%,<br>CUB-200: 54.7%,<br>SOP: 70.9%                                  |
|----|---|------|--|---|
| 20 | No Fuss Distance Metric Learning Using Proxies [70]   | 2017 | Product or image re-<br>trieval                            | CARS196: 73:22%<br>CUB200: 73.22%<br>SOP: 73.73%                                  |
| 21 | Sampling Matters in Deep<br>Embedding Learning [64]   | 2017 | Product or image re-<br>trieval,<br>Face Re-identification | CARS196: 86.9%<br>CUB200: 63.9%<br>SOP: 72.7%                                     |
| 22 | Deep Metric Learning via<br>Facility Location [99]  | 2017 | Product or image re-<br>trieval                            | CARS196: 58.11%<br>CUB200: 48.18%<br>SOP: 67.02%                                  |
| 23 | Deep spectral clustering<br>learning [53]   | 2017 | Product or image re-<br>trieval                            | CARS196: 73.07%<br>CUB200: 43.22%<br>SOP: 67.59%                                  |
| 24 | Hard-Aware Deeply Cascaded Embedding [133]  | 2017 | Product or image retrieval                                 | CARS196: 83.8%<br>CUB-200: 60.7%<br>In-shop: 62.1 %<br>SOP: 70.1%                 |
| 25 | PPFNet: Global Context<br>Aware Local Features<br>for Robust 3D Point<br>Matching [17]                            | 2018 | 3D Point Cloud<br>matching                                 | SUN3D: 71%  |
| 26 | Ranked List Loss for Deep<br>Metric Learning [121]  | 2019 | Product or image re-<br>trieval                            | CARS196: 82.1%<br>CUB-200: 61.3%<br>SOP: 79.8%                                    |
| 27 | Multi-Similarity Loss with<br>General Pair Weighting<br>for Deep Metric Learning<br>[119]                         | 2019 | Product or image retrieval                                 | CARS196: 77.3%<br>CUB-200: 65.7%<br>In-Shop: 78.2%                                |
| 28 | A Simple and Effective<br>Framework for Pairwise<br>Deep Metric Learning [82]                                     | 2019 | Product or image re-<br>trieval                            | In-shop: 91.3%<br>CARS-196: 86.2%<br>CUB-200: 68.1%                               |
| 29 | Deep Metric Learning<br>Meets Deep Clustering:<br>An Novel Unsupervised<br>Approach for Feature<br>Embedding [73] | 2020 | Product or image re-<br>trieval                            | CUB200: 47.5%,<br>Car196: 42.6%   |
| 30 | Exponential triplet loss [110]  | 2020 | Face Reidentification, Image Reidenification               | VGGFace2: 85.7%<br>EMNIST: 86%<br>FMNIST: 93.1%<br>CIFAR10: 87.3%<br>MNIST: 99.6% |

The Quality Assessment (QA) criteria are as follows:

- QA1: Does the publication provide open-source implementation of a novel loss function or methodology?
- QA2: Has a publication achieved state-of-the-art results on the datasets it studies?
- QA3: Does the publication provide a theoretical proof of a novel loss function or methodology?
- QA4: Does the publication include an ablation study to test effects on the results of functional parts one by one?
- QA5: Does a publication have over 100 citations?

Table 7: Evaluation of quality of publications baset on criteria.

| No  | Title   | QA1     | QA2 | QA3 | QA4 | QA5 | Total |
|-----|---|---------|-----|-----|-----|-----|-------|
| 11  | A Discriminative Feature                              | Yes     | Yes | No  | Yes | Yes | 4     |
|     | Learning Approach for<br>Deep Face Recognition        |         |     |     |     |     |       |
|     | [128]   |         |     |     |     |     |       |
| 18  | In Defense of the Triplet                             | Yes     | Yes | No  | Yes | Yes | 4     |
|     | Loss for Person Re-                                   |         |     |     |     |     |       |
| 21  | Identification [33] Sampling Matters in Deep          | No      | Yes | Yes | Yes | Yes | 4     |
| 21  | Embedding Learning [64]                               | No      | Yes | Yes | Yes | Yes | 4     |
| 23  | Deep spectral clustering                              | No      | Yes | Yes | Yes | Yes | 4     |
|     | learning [53]   |         |     |     |     |     |       |
| 28  | A Simple and Effective                                | Yes     | Yes | Yes | Yes | No  | 4     |
|     | Framework for Pairwise<br>Deep Metric Learning [82]   |         |     |     |     |     |       |
| 3   | Learning a Similarity                                 | No      | Yes | Yes | No  | Yes | 3     |
|     | Metric Discriminatively,                              |         |     |     |     |     |       |
|     | with Application to                                   |         |     |     |     |     |       |
| 5   | Face Verification [14] Large scale metric learn-      | No      | Yes | Yes | No  | Yes | 3     |
| э   | ing from equivalence con-                             | INO INO | res | res | No  | res | 3     |
|     | straints [46]   |         |     |     |     |     |       |
| 9   | FaceNet: A unified                                    | No      | Yes | No  | Yes | Yes | 3     |
|     | embedding for face                                    |         |     |     |     |     |       |
|     | recognition and<br>clustering [23]                    |         |     |     |     |     |       |
| 16  | Metric Learning with                                  | Yes     | Yes | No  | No  | Yes | 3     |
|     | Adaptive Density Dis-                                 |         |     |     |     |     |       |
|     | crimination [86]                                      |         |     |     |     |     |       |
| 17  | L2-constrained Softmax<br>Loss for Discriminative     | No      | Yes | No  | Yes | Yes | 3     |
|     | Face Verification [84]                                |         |     |     |     |     |       |
| 19  | Deep Metric Learning                                  | No      | Yes | Yes | No  | Yes | 3     |
|     | with Angular Loss [117]                               |         |     |     |     |     |       |
| 20  | No Fuss Distance Met-                                 | No      | Yes | Yes | No  | Yes | 3     |
|     | ric Learning Using Proxies [70]                       |         |     |     |     |     |       |
| 22  | Deep Metric Learning via                              | No      | Yes | Yes | No  | Yes | 3     |
|     | Facility Location [99]                                |         |     |     |     |     |       |
| 25  | PPFNet: Global Context                                | No      | Yes | No  | Yes | Yes | 3     |
|     | Aware Local Features<br>for Robust 3D Point           |         |     |     |     |     |       |
|     | Matching [17]   |         |     |     |     |     |       |
| 27  | Multi-Similarity Loss with                            | Yes     | No  | No  | Yes | Yes | 3     |
|     | General Pair Weighting                                |         |     |     |     |     |       |
|     | for Deep Metric Learning [119]                        |         |     |     |     |     |       |
| 1   | Signature Verification Us-                            | No      | Yes | No  | No  | Yes | 2     |
| 1   | ing A "Siamese" Time De-                              | '''     | 103 | 110 | 110 | 103 | -     |
|     | lay Neural Network [7]                                |         |     |     |     |     |       |
| 4   | Distance metric learning                              | No      | No  | Yes | No  | Yes | 2     |
|     | for large margin near-<br>est neighbor classification |         |     |     |     |     |       |
|     | est neighbor classification [127]                     |         |     |     |     |     |       |
| 6   | Quadruplet-Wise Image                                 | No      | No  | Yes | Yes | No  | 2     |
|     | Similarity Learning [52]                              |         | 1   |     | 1   |     |       |
| 7   | Reidentification by Rela-<br>tive Distance Comparison | No      | Yes | No  | No  | Yes | 2     |
|     | [135] tive Distance Comparison                        |         |     |     |     |     |       |
| 8   | Deep Metric Learning                                  | No      | No  | Yes | Yes | Yes | 2     |
|     | for Practical Person                                  |         | 1   |     |     |     |       |
| 1.5 | Re-Identification [132]                               |         | .,  | 1   | 1,  | 1,  |       |
| 10  | Improved Deep Metric<br>Learning with Multi-class     | No      | No  | No  | Yes | Yes | 2     |
|     | N-pair Loss Objective [98]                            |         |     |     |     |     |       |
| 12  | Deep Metric Learning via                              | No      | Yes | No  | No  | Yes | 2     |
|     | Lifted Structured Feature                             |         | 1   |     |     |     |       |
|     | Embedding [100]                                       |         |     |     |     |     |       |
|     |   |         |     |     |     |     |       |

| 14 | Learning Deep Embeddings with Histogram Loss [113]  | Yes | No  | No  | No  | Yes | 2 |
|----|---|-----|-----|-----|-----|-----|---|
| 24 | Hard-Aware Deeply Cas-<br>caded Embedding [133]   | Yes | No  | No  | No  | Yes | 2 |
| 26 | Ranked List Loss for Deep<br>Metric Learning [121]  | No  | Yes | No  | Yes | No  | 2 |
| 29 | Deep Metric Learning<br>Meets Deep Clustering:<br>An Novel Unsupervised<br>Approach for Feature<br>Embedding [73] | No  | No  | Yes | Yes | No  | 2 |
| 30 | Exponential triplet loss [110]  | Yes | Yes | No  | No  | No  | 2 |
| 2  | Neighbourhood Compo-<br>nents Analysis [27]   | No  | No  | No  | No  | Yes | 1 |
| 13 | Deep<br>clustering: Discriminative<br>embeddings for segmenta-<br>tion and<br>separation [34]                     | No  | No  | No  | No  | Yes | 1 |
| 15 | Local Similarity-Aware Deep Feature Embedding [37]  | No  | No  | No  | No  | Yes | 1 |

After reviewing over 30 publications in the field of DML, the following answers have been found to the research questions (RQ):

- RQ1: In this study, 27 types of loss functions for DML have been identified. They have been categorized and listed in their historical order in Fig. 4. All the DML loss functions originate from Margin Ranking Loss, which itself is a variant of earlier Hinge Loss functions [127]. Then most of the newer loss functions originate from Contrastive loss [7], Triplet Loss [23], Histogram Loss [113], and Quadruplet Hinge Loss [52]. For most of the publications included in the study research subject is either the loss function itself or the sample mining methodology.
- RQ2: Latest loss functions and sample mining strategy achieve significantly better results than the previous functions as seen in Table
   Also, datasets used in experiments have changed over time, but practical applications like image re-identification have not.
- RQ3: Most of the novel loss functions do not have theoretical explanations or derivations of the novel loss functions used in the model, but nonetheless some loss functions like Contrastive Loss [14], Triplet Hinge Loss [127], KISS-BCE Loss [46], Quadruplet Hinge Loss [52] and Binomial Deviance Loss [132] do have theoretical proof. Most of the other loss functions discussed in this study have their grounding in empirical experiments.

• RQ4: A number of significant limitations of DML loss functions and methods have been found in this study. Most of the loss functions require hyper-paramter  $\alpha$  that is, a margin between clusters, but in realistic datasets this might not be equal for all classes. Some classes might have more variance than others. Some efforts have been made to resolve the issue like Proxy NCA Loss [70], but even this loss function requires hyper-parameter tuning and prior knowledge of class distributions.

Intra-class similarities are also a significant problem. Most of DML loss functions ignore the fact that the same class samples also have their own distributions of similarities. Some works address this problem, but it still not fully solved [86] [52] [121].

Sample mining strategies also are a major problem as they require significant computing resources dedicated just for selecting the best samples to train the model and apply the loss function. Multiple sampling strategies have been developed like Hard [23], Semi-Hard [23], N Hard Mining [98], Neighborhood Sampling [86], Distance weighted sampling [64] and others, but the problem is still not yet fully solved.

Choice of the number of dimensions of embedding vectors and their embedding space also is a problem that still needs more studies. Publications differ in suggestions, how many dimensions to choose, and what normalization methods to apply to embeddings. Typical method is to use Euclidean distance with L2 normalized embeddings with high dimensionality of at least 128 dimensions [23], but some of the latest papers propose also alternative embedding space normalization [37] [117] and lower number of dimensions per embedding [110].

Another significant limitation is the computing resources required to reach higher accuracy in re-identification tasks, some earlier works from 2015 required over 2000 CPU hours to reach the highest accuracy on face re-identification tasks [23]. Latest works have been using GPUs to accelerate and parallelize training, but even nowadays as datasets grow larger that requires expensive GPU hardware [110].